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2002 J. Phys. A: Math. Gen. 35 2755

(http://iopscience.iop.org/0305-4470/35/12/305)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **35** (2002) 2755–2760 PII: S0305-4470(02)29054-X

Short-time scaling in the critical dynamics of an antiferromagnetic Ising system with conserved magnetization

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Received 19 September 2001, in final form 7 February 2002
Published 15 March 2002
Online at stacks.iop.org/JPhysA/35/2755

Abstract

We study by Monte Carlo simulation the short-time exponent θ in an antiferromagnetic Ising system for which the magnetization is conserved but the sublattice magnetization (which is the order parameter in this case) is not. This system belongs to the dynamic class of model C. We use nearest-neighbour Kawasaki dynamics so that the magnetization is conserved *locally*. We find that in three dimensions θ is independent of the conserved magnetization. This is in agreement with the available theoretical studies, but in disagreement with previous simulation studies with a global conservation algorithm. However, we agree with both these studies regarding the result $\theta_C \neq \theta_A$. We also find that in two dimensions, $\theta_C = \theta_A$.

PACS numbers: 05.50.+q, 05.10.Ln, 75.10.−b

In equilibrium statistical physics, universal scaling laws are observed close to a critical point where the correlation length diverges. Dynamical systems also exhibit a universal scaling behaviour in the long-time regime. Dynamic universality classes are characterized by the dynamic exponent, which connects the divergences in space and time. Typically, in a magnetic system, the finite-size scaling form of a physical observable $O(t, \tau, L)$ is given by,

$$O(t, \tau, L) = b^{-x} O(b^{-z}t, b^{1/\nu}\tau, b^{-1}L) \quad (1)$$

where $\tau = (1 - T/T_c)$ is the deviation from the critical temperature T_c, b the scaling factor, ν and x the static critical exponents, z the dynamic exponent and L the linear size of the system.

Some time ago, it was found that there is a universal behaviour in the short-time regime as well [1]. If a magnetic system is quenched from a high temperature to its critical temperature T_c, with initial order parameter equal to m_0, then universal scaling behaviour is observed in a macroscopic short-time regime:

$$M(t, \tau, L, m_0) = b^{-\beta/\nu} M(b^{-z}t, b^{1/\nu}\tau, b^{-1}L, b^{x_0}m_0) \quad (2)$$

where M is the order parameter, t is time, β is the critical exponent associated with the order parameter and x_0 is a new exponent associated with the short-time effect. x_0 is the scaling dimension of m_0. At the critical point, for *small* values of m_0,

$$M(t, m_0) \sim m_0 t^{\theta} \tag{3}$$

with $\theta = (x_0 - \beta/\nu)/z$. θ is a new exponent, not related to any previously known static or dynamic exponent [1]. The phenomenon is universal, as θ does not depend on microscopic details, updating schemes etc. It is observed in a macroscopic short-time regime.

A positive value of θ will indicate that the magnetization (or order parameter in general) will first increase with time and later show the conventional power-law decrease. It was observed in many systems that such an increase does indeed occur. The exact cause of this behaviour is not yet known very clearly.

Systems with different critical dynamics have been classified as models A, B, C etc [2]. Model A has no conservation, in model B the order parameter is conserved and in model C the non-conserved order parameter is coupled to a non-ordering conserved field. These dynamic classes are distinguished by the different values of the dynamic exponent z.

Using field theoretical methods, it was shown in [1] that short-time behaviour exists in model A. Extensive numerical studies calculating θ in the different dynamical classes have also been made in recent years and accurate estimates of θ in model A are available in different dimensions [3]. The results confirm the qualitative behaviour predicted by the theoretical analysis. Systems belonging to the class of model B do not show any short-time effect. However, numerical studies with a globally conserved order parameter in two dimensions indicate that there could be a short-time effect [4].

In model C, field theoretic techniques have shown that there is a universal short-time behaviour [5]. Short-time effect in model C in a semi-infinite geometry has also been studied [6]. There have been some recent numerical studies of the short-time scaling in model C with global conservation [7]. These numerical studies show the novel result that the short-time exponent depends on the value of the globally conserved magnetization—a feature not previously obtained, at least in the theoretical study with local conservation. However, this is not very surprising, as systems with global conservation in general show different behaviour compared to the ones with local conservation [8–11] as far as the long-time behaviour is concerned. In model C and model B for example, the behaviour with global conservation becomes model-A-like.

However, before any conclusive statement about differences in short-time behaviour in model C for local and non-local conservation can be made, it is necessary to obtain numerical estimates of the exponents for the cases where the coupling field is locally conserved. This is because the numerical estimates and field theoretic results could be quite different below the upper critical dimension. In this paper, we report estimates of the short-time exponents in both two and three dimensions with several values of the conserved density from simulation studies with local conservation.

We have taken an antiferromagnetic Ising system where the magnetization m_0 is kept constant. This model has been studied numerically previously to estimate the dynamic exponent z [12] and the short-time exponent θ with global conservation [7]. The order parameter in this system is the staggered magnetization m_s which is not conserved. Initially the system is allowed to have short-range correlations only (since we quench it from a high temperature to T_c) and the initial $m_s(t = 0)$ is kept at a low value. Hence the system is prepared with two constraints: both m_0 and m_s are kept fixed. This could be done by keeping a staggered field, but we simply keep it at the desired value by appropriately flipping spins in a random configuration (the method is analogous to what is called a sharply

Table 1. Numerical estimates of the critical temperature T_c for different values of the conserved magnetization m_0. The estimates for $d = 3$ with $m_0 \neq 0$ are in agreement with [7].

Dimension	Size of lattice	m_0	$k_B T_c$
2	1001×1002	0	2.2692[a]
		0.1	2.108 ± 0.002
		0.2	1.557 ± 0.002
3	$101 \times 101 \times 102$	0	4.5116[b]
		0.08	4.453 ± 0.004
		0.12	4.376 ± 0.004
		0.2	4.142 ± 0.004
		0.4	3.009 ± 0.004

[a] Exact result.
[b] Approximate value; see e.g. [7].

prepared state in [3]). The antiferromagnetic interactions are between nearest neighbours. The lattices are hypercubic. Periodic boundary conditions along one direction and helical boundary conditions along the other directions are used. Helical boundary conditions with antiferromagnetic nearest-neighbour interactions demand that the linear size along $(d - 1)$ directions should be odd where d is the spatial dimension.

We use the Kawasaki dynamics where the opposite spins on nearest-neighbouring sites are exchanged, thus keeping m_0 constant. The exchange between local neighbours ensures that the conservation is local. The spins are updated sequentially and a sweep through the entire system is equivalent to one unit of time (one Monte Carlo step). In both two and three dimensions, we fix m_0 at several different values. Since the system is quenched to the critical temperature $T_c(m_0)$, we first estimate the value of $T_c(m_0)$ (table 1) for a large system following the method used in [12]. We then estimate θ for each value of m_0 in both two and three dimensions in smaller lattices. For this estimate, one measures the initial slope of the $m_s(t)$ versus t curve for different values of $m_s(0)$ and takes the limit $m_s(0) \to 0$.

The results are as follows. In *two dimensions*, the short-time exponent θ has been found to be

$$\theta = 0.18 \pm 0.01 \tag{4}$$

and this value has no measurable dependence on the initial (small) value of the sublattice magnetization $m_s(0)$ (figure 1) or on the conserved magnetization m_0 (figure 2). Also, in contrast with the simulation studies with the global conservation algorithm [7], it is the same for models A and C, up to the accuracy of the present study. In *three dimensions*, the exponent θ has been found to be

$$\theta_C = 0.13 \pm 0.01 \tag{5}$$

and this value also has no measurable dependence on the initial (small) value of the sublattice magnetization $m_s(0)$ (figure 3). In agreement with the theoretical predictions [5] and in contrast with the results of the simulation studies with the global conservation algorithm [7], this value of θ has no numerically detectable dependence on the conserved magnetization m_0 (figure 4) at least for $m_0 \geqslant 0.12$. However, in agreement with both the studies [5, 7], the exponent has a different value for model A:

$$\theta_A = 0.10 \pm 0.01. \tag{6}$$

There is some indication that for very small values of m_0 ($\leqslant 0.08$), θ might depend on m_0 (figure 4) but a detailed study in this region requires very large-scale simulations.

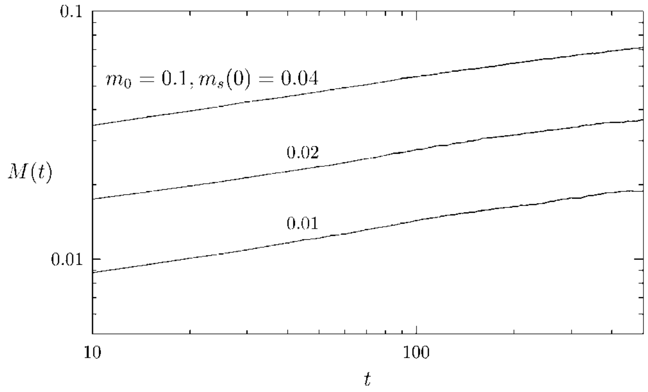

Figure 1. The early-time effect at critical temperature in two dimensions for a fixed value of the conserved magnetization m_0, and various values of the initial sublattice magnetization $m_s(0)$. The lattice size was 349 × 350 and the results were averaged over 20 000–50 000 realizations of the system.

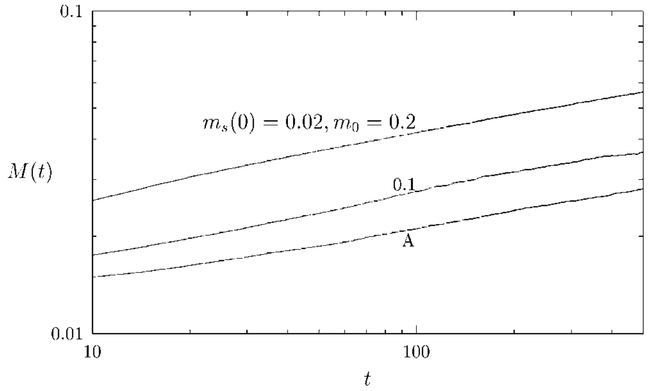

Figure 2. The early-time effect at critical temperature in two dimensions for a fixed value of the initial sublattice magnetization $m_s(0)$ and two values of the conserved magnetization m_0. The line marked A is for model A, i.e. $m_0 = 0$. The lattice size was 349 × 350 and the results were averaged over 20 000–50 000 realizations of the system.

A few comments are in order.

(i) Our estimate of θ in two dimensions for model A is in agreement with previous estimates [3]. The estimates for model A are obtained by putting $m_0 = 0$ in the present model (as $m_0 = 0$ corresponds to model A [13]).

(ii) That $\theta_A = \theta_C$ in two dimensions indicates that the conservation is irrelevant here. This is because the specific heat exponent α is negative here [2, 5], and the estimate of θ_C given by [5] is strictly true for $2 < d < 4$ when the spin dimension $n = 1$.

(iii) In three dimensions, our result $\theta_A \neq \theta_C$ and θ_C being independent of m_0 are in qualitative agreement with the theoretical estimates [5].

(iv) As regards the dependence of θ_C on m_0 in three dimensions, the discrepancy between our results and that of [7] is not surprising, because non-local conservation may be expected to give new values of exponents and can even change the universality classes. In fact, the numerical estimate of the dynamical exponent z in the globally conserved model C [7] is found to correspond to that of model A, a result predicted by the analytical study of [10].

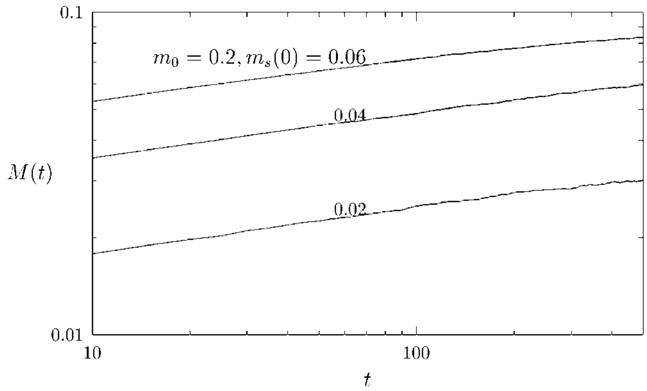

Figure 3. The early-time effect at critical temperature in three dimensions for a fixed value of the conserved magnetization m_0, and various values of the initial sublattice magnetization $m_s(0)$. The lattice size was $65 \times 65 \times 66$ and the results were averaged over 5000 realizations of the system.

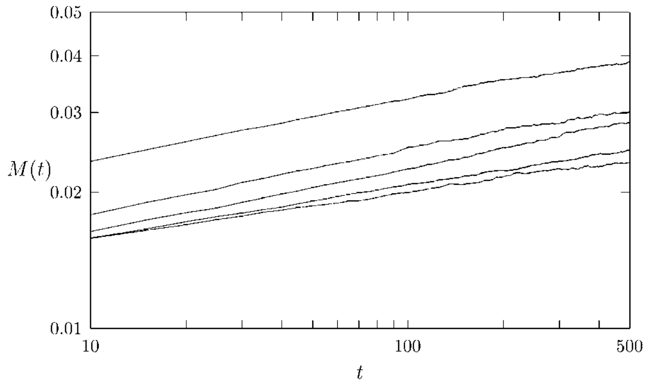

Figure 4. The early-time effect at critical temperature in three dimension for a fixed value of the initial sublattice magnetization $m_s(0)$ and several values of the conserved magnetization m_0. From top to bottom, the lines correspond to $m_0 = 0.4, 0.2, 0.12, 0.08, 0$ (model A); the results were averaged over 5000, 5000, 50 000, 50 000, 13 000 realizations, respectively. The lattice size was $65 \times 65 \times 66$ in all cases.

(v) The results shown in the figures are for the largest sizes simulated. Simulation for smaller sizes, e.g., 199×200 in two dimensions and $41 \times 41 \times 42$ in three dimensions show that there is no detectable finite-size effect.

(vi) There could be some error in the estimate of θ due to the error in the estimation of T_c. We have, however, checked that θ does not vary measurably when T_c is varied about its estimated value within the error bar.

Lastly, we should mention that one can also estimate θ from the behaviour of the autocorrelation function

$$A(t, 0) = \langle S_i(t) S_i(0) \rangle - \langle S_i(t) \rangle \langle S_i(0) \rangle \sim t^{-\lambda} \tag{7}$$

where $\lambda = d/z - \theta$. In this method, one can put $m_s(0) = 0$. One needs to know z very accurately to get an accurate estimate of θ using this method. Also, a good estimate of θ requires lattice sizes much greater than the ones simulated in the present study. Nevertheless, we could calculate $A(t, 0)$ and verify that λ (and hence θ) is indeed independent of m_0.

Acknowledgments

The authors are grateful to S M Bhattacharjee for valuable discussions and suggestions and to B Zheng for useful comments. The computational facilities were provided by CUCC. PS acknowledges financial support from DST project SP/S2/M-11/99.

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